Modeling Expectations with GENEFER

- an Artificial Intelligence approach

by

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June 2000

Abstract

Economic modeling of financial markets attempts to model highly complex systems in which expectations can be among the dominant driving forces. It is necessary, then, to focus on how agents form expectations. We believe that they look for patterns, hypothesize, try, make mistakes, learn and adapt. Agents' bounded rationality leads us to a rule-based approach which we model using Fuzzy Rule Bases. For example if a single agent believes the exchange rate is determined by a set of possible inputs and is asked to state his relationship, his answer will probably reveal a fuzzy nature like: "*IF the inflation rate in the EURO-Zone is low and the GDP growth rate is larger than in the US THEN the EURO will rise against the USD*". '*Low'* and '*larger'* are fuzzy terms which give a gradual linguistic meaning to crisp intervalls in the respective universes of discourse. In order to learn a Fuzzy Fuzzy Rule base from examples we introduce Genetic Algorithms and Artificial Neural Networks as learning operators. These examples can either be empirical data or originate from an economic simulation model. The software *GENEFER* (GEnetic NEural Fuzzy ExploreR) has been developed for designing such a Fuzzy Rule Base. The design process is modular and comprises Input Identification, Fuzzification, Rule Base Generating and Rule Base Tuning. The two latter steps make use of genetic and neural learning algorithms for optimizing the Fuzzy Rule Base.

1. Introduction

Modeling expectations is a major endeavor for economists as well as for psychologists. In economics there are a variety of theories requiring explicit expectation modeling. Here we deal with financial markets because of the predominant influence of forecasts on asset market transactions. Since financial markets display phenomena such as bubbles, crashes, herd behavior, contagion, GARCH effects, or focal points, we ask: What moves the asset prices? FARMER (1999) gives a first hint by stating: "... to have a good theory of how prices behave, we will need to explain the behavior of the agents on whom they depend". The standard academic literature is still not very convincing about modeling expectation formation. Although agents face a pool of publicly available information, which consists of past prices, trading volumes, economic indicators, political events, rumors, news, etc., "there may be many different, perfectly defensible statistical ways based on different assumptions and different error criteria to use them…" [ARTHUR (1995)]. That is the point where the psychologists' view comes into play. Some cognitive scientists believe that agents form 'mental models' of the world in order to deal with complex environments [see Johnson-Laird (1983)]. They look for patterns, hypothesize, try, make mistakes, learn and adapt [MARENGO/TORDJMAN (1996), p. 410]. In doing so they inductively form expectations [ARTHUR (1994)]. The attempt to model their underlying mental processes explicitly is challenging, but it "… helps to ensure that theorists are not taking too much for granted and that their theories are not vague, incoherent, or, like mystical insights, only properly understood by their proponents" [JOHNSON-LAIRD/SHAFIR (1993)]. In recent years, there is a growing literature about the use of Artificial Intelligence (AI) methods for modeling these 'mental models' and their adaptation to a constantly evolving economic environment. Several papers are about John Holland's Classifier Systems [see HOLLAND (1975)] and their applications to various economic problems [see VRIEND (2000), ARTHUR ET AL. (1996), MARENGO/TORDJMAN (1996) and BELTRAMETTI ET AL. (1997)]. As discussed in section 2, this technique does not meet all the demands placed on an AI-based approach for modeling expectations. Therefore, we propose an alternative way to model the formation of expectations in a complex environment using Fuzzy Rule Bases (FRB) as an operational representation of 'mental models'. We divide the design process of such an FRB into four major steps and apply Genetic Algorithms as well as Artificial Neural Networks to learn and to tune FRBs from observed data. In section three these steps of modeling and training FRBs are described. We do so by following a typical FRB-design session with the software tool GENEFER (GEnetic NEural Fuzzy ExploreR), which was developed for handling Neural

and Genetic Fuzzy Rule Bases and whose key features for model builders are highlighted in section 3.6. The paper closes with a glance at possible applications of GENEFER and some suggestions for further research.

2. A Genetic and Neural Fuzzy Rule Base approach towards modeling expectations

The 'true model' of the world economy or national economies or even their single subsystems (e.g. markets, industries or firms) has yet to be found. The massive interaction between millions of heterogeneous agents produces this complexity, which is continuously challenging our understanding of the world we live and (trans)act in. Although we have recorded stock and bond prices, interest rates, exchange rates and many more prices for more than a century, we still lack a commonly accepted theory explaining the formation of these prices. This is especially so in financial-market research, where we see several competing theories that seem appropriate to explain specific market phenomena. Recent research breaks away from the widespread idea of neoclassical market foundations and the assumption of homogenous rational agents who instantaneously discount new information into prices, so that no technical trading can offer any consistent speculative profits and cause markets to appear to be perfectly efficient. Homogeneity assumes mutually consistent perceptions about the environment (one commonly shared 'true model') and allows for the representative agent framework. There is accumulating behavioral evidence against this rational view[1] as well as theoretical objections including the costs of gathering and processing information, the costs of transacting, bounded rationality of agents, and indeterminacy of deductively formed expectations [ARTHUR (1995), p. 8; ALBIN (1998)].

Real financial markets are characterized by heterogeneous agents who have different motives to trade, different planning horizons [OLSEN ET AL. (1992)] and different beliefs about the 'functioning or driving forces of the market' and future events that affect their action today and influence asset prices tomorrow. Financial decision makers face theoretical (competing theories), empirical (spurious correlation), and operational (non-measurability of inputs) difficulties in identifying relevant input data. Therefore, they do not base their decisions on a uniform input data set or on a single accepted theory, but rather on a mixture of theories, technical analyses (chart analyses), what their competitors do, and the like. Despite all these problems agents have to form expectations when (trans)acting in financial markets. Since their expectations help strongly to determine the aggregate market outcome they try to predict, expectations become self-referential, and a rational deduction of the 'true model'

[1] See McFadden (1999), p. 75 onwards. George Soros, one of the most successful and therefore most famous traders, expressed his opinion about the standard academic theory as follows: "…this [efficient market theory] interpretation of the way financial markets operate is severely distorted. ... It may seem strange that a patently false theory should gain such widespread acceptance".

becomes impossible. An agent cannot deductively form his expectations, since he needs to know others' (privately held) expectations to form his own – consequently these are indeterminate [FOLEY (1998), p. 53 ff; ARTHUR (1995), p. 3 ff]. Even without informational problems, an agent might not be able to understand fully the complexity of a market or if the costs of discovering the exact input-output relation in finite time exceed the benefits from it. Both limited information-processing capability stemming from the intrinsic data complexity and a lack of information bound the agents' rationality. When others' expectations are non-measurable, there is an absolute barrier to rationality. But, of course, agents form expectations everyday. How do they do it?

As mentioned, one theory holds that agents look for patterns or rules within the market. In doing so they mentally form a rule-based representation of the market's functioning. These rules reflect the agents' experiences and may be influenced by economic theory. This rule-based approach includes the expression of explicit (e. g. technical trading rules) as well as tacit knowledge.

Due to agents' bounded rationality, however, these rules should not be interpreted as exact mathematical functions but rather as a relationship between the agents' interpretation of the input and output variables' states. We contend that agents interpret these crisp states vaguely by associating with them a limited number of linguistic terms (e. g., 'low', 'medium', 'high').[2]

An evolving economic environment continuously generates new observations that extend an agent's experience. New observations can conflict with current knowledge so that it becomes necessary to learn by changing rules or the interpretation of crisp data.

From the preceding, we are led to the following list of demands on a realistic approach for modeling expectations:

(D1) explicit knowledge representation (theory-driven rules, model building)

(D2) vague formulation of forecasts (bounded rationality)

(D3) dependency upon experiences (ability to learn)

To meet these demands we choose Fuzzy Rule Bases as an operational representation of mental models and apply Genetic Algorithms (GA) and Artificial Neural Networks (ANN) as learning operators.

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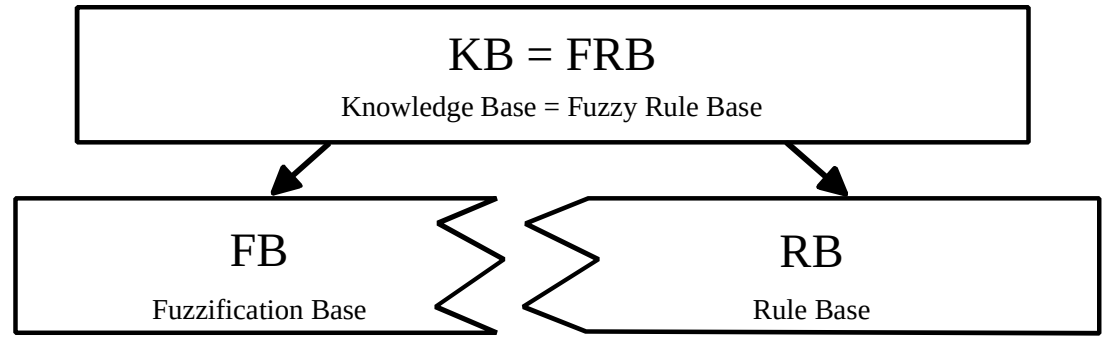
[2] This is opposed to classifier systems which usually interpret crisp data by means of crisp intervals.

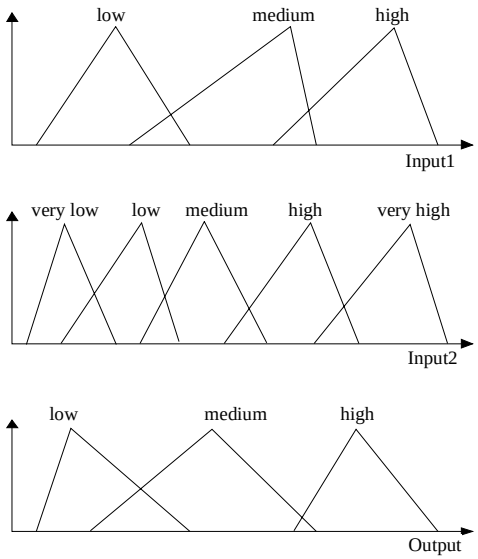
A single fuzzy rule expresses the vague relation between input(s) and output, like

> *IF the US current account deficit is very high AND the GDP growth rate is lower than in the Euro-Zone THEN the EURO is expected to rise strongly against the USD.*

The IF elements of the fuzzy rule are called antecedents, whereas the THEN elements are consequents. 'Very high', 'lower' and 'strongly' are fuzzy terms. The set of all fuzzy rules represents an agents knowledge base (KB) and is called Fuzzy Rule Base (FRB). An FRB can be divided into two parts, the rule-base RB and the fuzzification base FB (see Figure 1).[3] The former captures all rules as IF-THEN statements, whereas the latter provides the fuzzy sets, which express the terms' linguistic meaning as membership functions. These functions map the elements in the universes of discourse to the unit interval and allow for partial quantification of belongingness (e.g., a temperature of 20° C belongs to the fuzzy term 'warm' to a degree of 0.8). We additionally assign a weight to each rule indicating its relative importance within the RB and therefore allow for a simple hierarchy.

Figure 1: Structure of Fuzzy Rule Bases





IF	Input1	AND	Input2	THEN	Output
	low		medium		medium
	low		very high		high
	medium		very low		low
	high		very high		high
	low		low		low
	high		medium		medium
	high		very low		medium
	medium		low		low
	medium		medium		medium
	high		medium		medium

[3] This separation is also realised in GENEFER's data handling, which allows for a greater flexibility of linking different FBs to different RBs.

Both, FB and RB can be modified during an agent's learning process. The GA approach requires a suitable encoding of these objects in order to apply its genetic operators (selection, mutation, crossover). Transforming the FRB into an equivalent Artificial Neural Network opens it up to neural learning techniques (such as error-backpropagation).

Whereas fuzzy systems account for (D1) and (D2), their combination with GA- and ANN-training techniques meets all three demands. The use of Genetic and Neural Fuzzy Rule Bases and their implementation in GENEFER is sketched in the next section.

3. Managing Fuzzy Rule Bases with GENEFER

GENEFER is software for designing an FRB of the Mamdani multiple-input-single-output type. The primarily technical literature about fuzzy control proposes numerous ways of setting up an FRB. GENEFER does not focus on one of the many ways but separates the design process in four major steps (and thus allows the user to combine alternative methods): (i) identifying inputs, (ii) fuzzifying crisp input and output data (FB), (iii) generating a rule base (RB) and finally (iv) tuning the FRB.[4] The design methods of each step can be run either in an inductive or in a manual (expert) mode. The former applies combinations of GA or ANN with Fuzzy Logic Controllers [LIN/LEE (1991), LIN (1994) and CORDÓN/HERRERA (1997)] and requires a training data set of observations (TDS) for learning. GENEFER is equipped with a DESIGN NAVIGATOR that allows quick and easy navigation through the process of FRB design. The description of each step below shows the respective DESIGN NAVIGATOR's appearance.

We will not explain the basics of fuzzy inference except to mention that we use t-norm operators for calculating the degree of activation AD_i of a single rule i and accumulate these using the sum procedure $[\min(1, \sum AD_i)]$ to obtain a compositional degree of activation. The fuzzy inference result can be defuzzified by the (i) Center of Maximum weighted by surface, (ii) Center of Maximum weighted by significance, (iii) Center of Gravity weighted by Surface, or (iv) Center of Gravity weighted by Significance. Since it is beyond our needs here, we also avoid giving details of GA and ANN and refer instead to the standard literature [e. g., PEDRYCZ/GOMIDE (1998)]. This paper is not intended as a user manual, but as means of highlighting the most crucial features for model builders to raise interest in applying GENEFER to economic simulations.

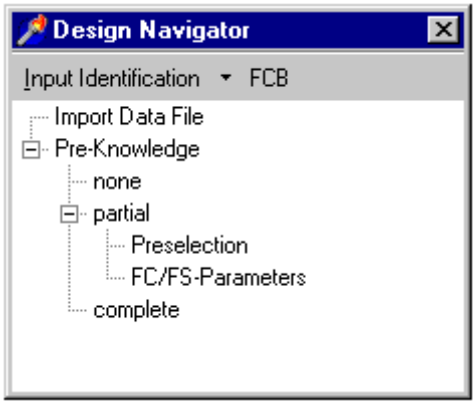
[4] The modular design is strongly influenced by the work of Cordón/Herrera (1997).

For better readability and easier access to the mathematical notation, we list our frequently used indexes and variables:

$j \in \{1,\ldots,J\}$	Input index	out_t	Output in t
$t \in \{1,\ldots,T\}$	Period index	A_i	Antecedent of Rule i
$i \in \{1,\ldots,M\}$	Rule index	A_{ij}	Fuzzy set of input j in Rule i
TDS	Training data set	B_i	Output fuzzy set in Rule i
$p \in \{1,\ldots,P\}$	Period index (only in 3.1.1.a)	$B = \{B_1,\ldots,B_{N_{Out}}\}$	Set of all output fuzzy sets
$n \in \{1,\ldots,N\}$	Fuzzy set index	$A_i(in_{jt})$	Membership value of in_{jt}
in_{jt}	Input j in t	$B_i(out_t)$	Membership value of out_t
$in_t = \{in_{1t},\ldots,in_{Jt}\}$	Input vector in t	$outFRB(in_t)$	Crisp inference result

3.1. Input Identification

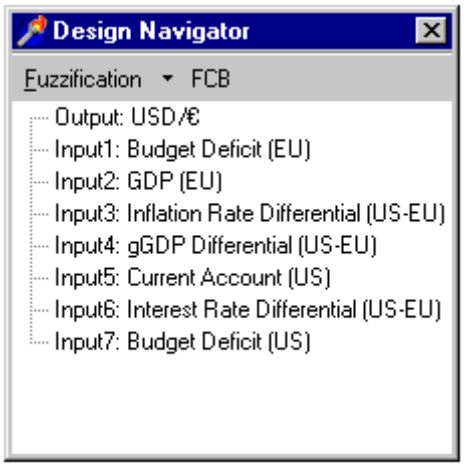
GENEFER's data interface uses an Excel 5 compatible file format with the following worksheet organization: horizontally the worksheet is divided into the output range (first column) and the input range (following columns). The first row is used for time series labels. Then come rows of numerical data equal in number of periods. After importing the data, GENEFER offers three ways to proceed. If the user has complete pre-knowledge of the relevant inputs, he can choose that option in the DESIGN NAVIGATOR and select the desired time series. Otherwise, he can apply GENEFER's identification algorithm. If the user is completely or partially ignorant about the set of independent inputs, he can proceed along the tree items 'none' and 'partial', respectively. In the latter case, preselected time series are definitely included in the final set of inputs before the user runs the FC/FS-identification algorithm. This algorithm eliminates unimportant and related inputs iteratively. In the first step it calculates J fuzzy curves (FC) for each input–output combination and determines the MSE between these curves and the observed output data. FC-Ranking in ascending order yields the most important input (lowest MSE). Each step is completed by eliminating a percentage β of insignificant inputs (highest MSE). The next step combines the previously identified input with each remaining possible input and output to calculate (1-β)·(N-1) fuzzy surfaces (FS). Sorting all FS according to MSE indicates the second most important input. Computation of these surfaces proceeds until the set of possible



inputs is empty. The FC/FS-algorithm is a computationally fast means for isolating an independent set of significant input variables of a complex, poorly defined, nonlinear system.[5]

3.2. Fuzzification

Once the previous procedure has been accomplished, a variable's fuzzification can be accessed directly via the DESIGN NAVIGATOR. Clicking on a variable opens the Fuzzification Dialog in Figure 2. Starting with a default FB, this dialog allows the user to fuzzify each variable separately. Neither the degree of granularity (number of fuzzy sets) nor the type of fuzzy set (triangular or gaussian) is applied uniformly but may differ across inputs and output. We know from cognitive science that human beings are typically capable of distinguishing up to 7 significant classes, so we restrict the number of fuzzy sets per variable to be either 3, 5 or 7 [ALTROCK (1995), p. 153, and PEDRYCZ/GOMIDE (1998), p. 67].



The fuzzification dialog assists the user in fuzzifying the output and the identified inputs easily. GENEFER displays the default fuzzification for each variable if the user has not specified a FB yet. The user may modify this by changing the settings in the displayed form in figure 2. The selected variable is in ①. The user determines the number of fuzzy sets in ② and their type in ③. Grid ④ shows the centers and widths but also allows for manual editing if this option is selected (deactivation of ⑤). Additionally, a clustering algorithm is offered whose settings can be found in ⑥. This algorithm runs over all periods in the fuzzification interval ⑦ and guarantees a minimum membership value (equal to the overlap in ⑥) for each observation of the selected variable within this interval. Maximum and minimum values of the chosen variable within this interval are displayed in ⑧. It is important to point to the 'S-Shape (Border Sets)' option ⑨. If a new observation is outside the support of all fuzzy sets, this option helps to avoid an inability to generate an inference result.[6]

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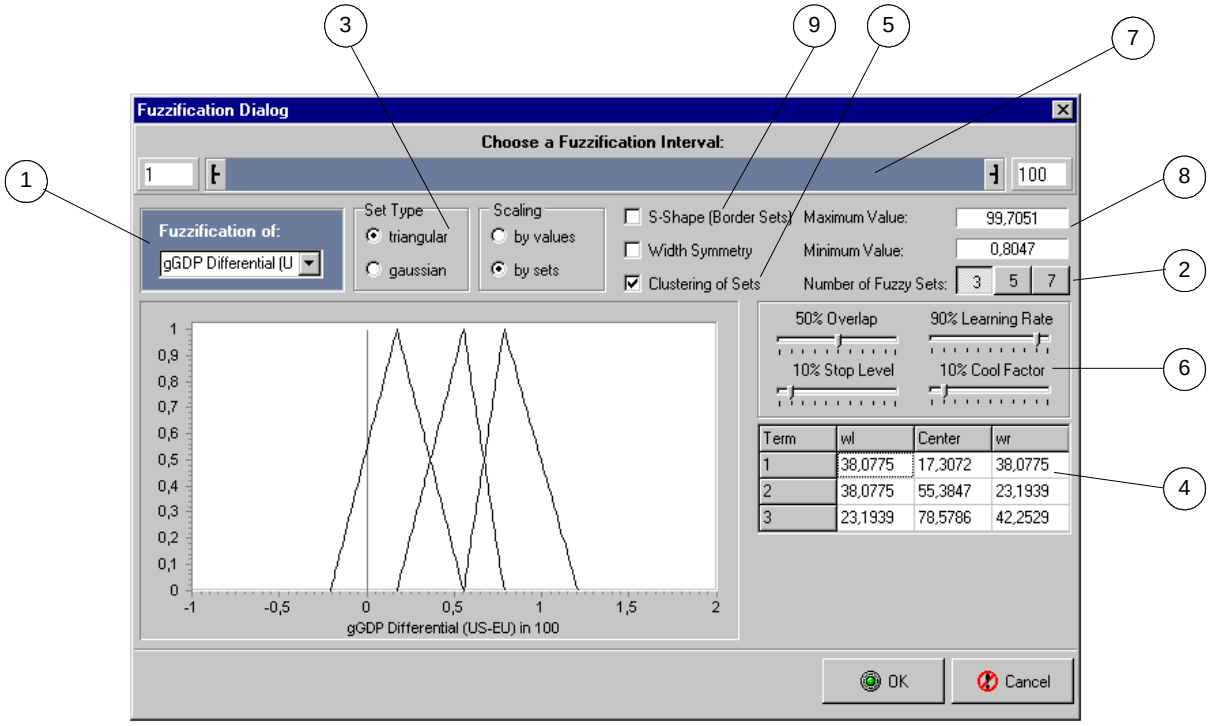
[5] For a detailed description of the FC/FS-algorithm see Lin/Cunningham/Coggeshall (1996), pp. 65-71. In addition to their proposal, all input time series are scaled within the unit interval in order to avoid distorted results due to significant differences in standard deviations.

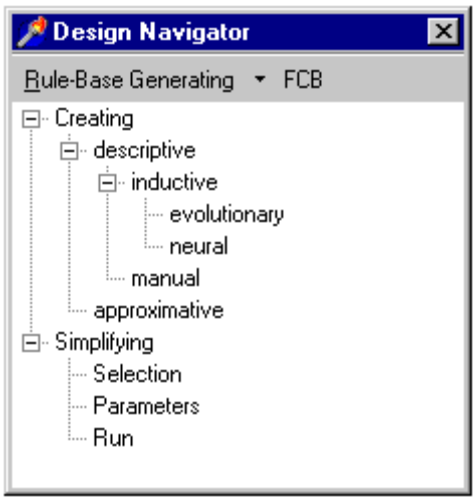
[6] Note that this failure cannot occur during the FRB-design process, since the interval for inductive learning can only be equal or part of the interval in the previous step.

Figure 2: Fuzzification Dialog



3.3. Rule Base Generating

The user's decision about the granularity of input variables determines the maximum number of possible rules $M_{Max} = \prod_{j=1}^{J} N_j$, which is equal to every possible combination of input fuzzy sets. An RB consisting of M_{Max} rules inevitably becomes intractable if the number of inputs increases. Therefore it is advantageous to remove (or, better, not to create) all redundant rules. For example, if an agent who distinguishes among 'low', 'medium' and 'high' inflation rates and 'weak', 'medium' and 'strong' growths of GDP has never experienced a combination of low inflation and strong growth, there is no reason to have a rule for this case [LEGRENZI/GIROTTO/JOHNSON-LAIRD (1993), pp. 38 ff.].[7] The DESIGN NAVIGATOR above shows the two major steps of the RB generating process, which are now described.



[7] For those, who might ask what to do, if a new observation begets an unexperienced input value combination, we refer to section 3.5.

3.3.1. Rule Base Creating

Rule Base Creating has a descriptive and an approximate approach. They differ in the linguistic meaning of each fuzzy set when interpreting fuzzy rules. The descriptive approach is characterized by a uniform meaning for all fuzzy sets in all rules. If the second set of the third input appears in more than one rule, it always has the same meaning (e.g. medium) and is represented by the same membership function. Changing this function consequently affects all rules that contain the respective fuzzy set. Hence there are no restrictions on the economic interpretability of a descriptive FRB. By contrast, the approximate approach allows fuzzy sets to differ from rule to rule (free semantics). Whereas this may have advantages concerning the goodness of fit, it is not amenable to an economic interpretation. We do not consider the approximate approach in the following, but concentrate on the interpretably descriptive one.[8]

The designer of an FRB has to pay attention to two properties to obtain good results [PEDRYCZ/GOMIDE (1998), ch. 10.6, and especially GORDÓN/HERRERA (1997), pp. 377-380]. The *completeness* property guarantees that the FRB is able to generate an inference result for each observation of input values in the TDS. The *completeness* property can be assigned to the FB and the RB. An FB is *complete* if the union of all fuzzy sets for each input variable covers the related universe of discourse. If it covers to a level of $\sigma \in [0,1]$, the FB is called *σ-complete*. An RB is called *complete* if it is able to generate an inference result for any input proposition. As mentioned above, this may lead to an unacceptably large number of rules if certain regions in the input space are not covered or can be excluded. Therefore we relax the RB *completeness* property by requiring each observation $d_t = (in_t, out_t)$ in the TDS to be covered to a degree of at least $CV(d_t) \geq \varepsilon > 0$. The Covering Value CV is calculated as follows:

(1) $A_i(in_t) = \min(A_{i1}(in_{1t}), \ldots, A_{iJ}(in_{Jt}))$

(2) $R_i(d_t) = \min(A_i(in_t), B_i(out_t))$

(3) $CV(d_t) = \bigcup_{i=1}^{M} R_i(d_t)$

(4) $CV(d_t) = \sum_{i=1}^{M} R_i(d_t)$

[8] We plan to test GENEFER's forecast abilities and will, of course, include the approximate approach to see if it shows better results than the descriptive one.

Equation 1 is the degree of compatibility between the i^{th} rule's antecedent and the observation in_t, whereas equation 2 holds for the entire rule i and the observation d_t. Equation 3 is the generic Covering Value. However, the iterative nature of the evolutionary procedure creating an RB (see below) requires the modification in equation 4.

The second property of an FRB is *consistency*. A Rule Base is called *consistent* if it does not contain contradictions. A contradiction arises if two or more rules have the same antecedents but not the same consequent. In FRBs there is a need to relax the *consistency* requirement due to fuzzy modeling. It is the essence of fuzzy modeling that a crisp observation may be consistent with more than one fuzzy rule, which may of course differ in their consequents. But this does not necessarily imply contradictions, just like there is no contradiction in stating that a person is tall to a degree of 0,9 and very tall to a degree of 0,25. To avoid contradictions in FRBs we apply the concepts of positive and negative observations. An observation is regarded as positive for a fuzzy rule if it matches its antecedent and consequent with a compatibility degree $R_i(d_p)$ greater than or equal to ω:

(5) $$TDS_{p,\omega}^{+}(R_i) = \{d_p \in TDS_p \mid R_i(d_p) \geq \omega\}.$$

If the antecedent matches but not the consequent the observation is considered negative for the rule:

(6) $$TDS_p^{-}(R_i) = \{d_p \in TDS_p \mid R_i(d_p) = 0 \text{ and } A_i(in_p) > 0\}.$$

With respect to this relaxation, we call an FRB *consistent* if it provides a sufficiently small number of negative observations, measured as the percentage k of positive observations.

GENEFER offers three options to create an FRB as shown in the DESIGN NAVIGATOR above. The two first create an FRB inductively by using the information inside the TDS. The third allows for the manual creation of fuzzy rules to make use of expert knowledge.

a) evolutionary

The evolutionary method is an iterative process that generates a complete and consistent RB for all observations in the TDS_p. It is therefore necessary to select these observations carefully so that they cover all possible input combinations. One step encompasses the four following substeps:

- Creation of a candidate RB matching all observations within the TDS_p,
- Evaluation of all candidate rules according to a fitness function,
- Copying the best (fittest) rule to the generated RB and clearing the candidate RB,

- Removal of all observations in the TDS_p for which $CV(d_t) > \varepsilon$.

A candidate rule is created by linking those fuzzy sets that yield the highest membership value for the values of the variables of the current observation. A TDS_p of P observations therefore leads to a candidate RB of P or less fuzzy rules, since doubles are excluded. The consecutive evaluation uses a multicriterion fitness function that considers the following criterions:

(7) *High Frequency Value:*

$$\psi_{TDS}(R_i) = \frac{\sum_{p=1}^{P} R_i(d_p)}{P}$$

(8) *High Average Covering Degree Over Positive Observations:*

$$G_\omega(R_i) = \sum_{d_p \in TDS_{p,\omega}^{+}(R_i)} \frac{R_i(d_p)}{|TDS_{p,\omega}^{+}(R_i)|}$$

(9) *Small Negative Example-Set:*

$$g_n(R_i^{-}) = \begin{cases} 1 & \text{if } |TDS_p^{-}(R_i)| \le k \cdot |TDS_{p,\omega}^{+}(R_i)| \\ \dfrac{1}{|TDS_p^{-}(R_i)| - k \cdot |TDS_{p,\omega}^{+}(R_i)| + \exp(1)} & \text{otherwise} \end{cases}$$

(10) *Fitness function:*

$$F(R_i) = \psi_{TDS}(R_i) \cdot G_\omega(R_i) \cdot g_n(R_i^{-})$$

The best rule, the one with the highest fitness value, is copied to the generated RB. This generated RB is then used to compute the Covering Value for all $d_p \in TDS_p$ (see (4)), and all observations whose CV is greater than or equal to ε are removed. The candidate RB is cleared for the next step, which runs over the reduced TDS_p. This process terminates when $TDS_p = \varnothing$.

b) neural

As an alternative to the evolutionary procedure GENEFER offers an unsupervised competitive neural learning algorithm for detecting rules in a given set of observations.[9] The

[9] For the underlying feature-map-algorithm see Kohonen (1988), ch. 5, particularly p. 132.

algorithm works as follows: At the beginning of the learning process all antecedents i are "virtually connected" with all consequent terms. Virtually connected means that there is a potential connection between each antecedent i ($i = 1,...,M_{Max}$) and each consequent set q ($q = 1,...,N_{Out}$) with an initial connection weight of zero ($w_{iq} = 0 \; \forall$ i). Presenting a training pattern d_t allows computation of the degree of activation for each antecedent $A_i(in_t)$ and the membership value for each fuzzy set q of the output variable $B_q(out_t)$. These values are used to adapt the connection weights by means of the following learning rule:

$$\Delta w_{iq} = B_q(out_t) \cdot [-w_{iq} + A_i(in_t)]. \tag{11}$$

At the end of the learning procedure (after a given fixed number of training patterns) the connection with the maximum connection weight of each antecedent is kept and all others are removed.

c) manual

GENEFER's expert mode allows one to (re-)design the complete RB or parts of it manually, including the rule-weights vector (setting a rule-weight to zero deactivates the rule). If no RB has yet been created, the program delivers an initialized normalized RB by generating all M_{Max} antecedents with the output's mid-term as the default consequent. This basic RB can then be used as a starting point for further individual modeling.

By setting influence factors $inff_j$ for each input variable j, the antecedents of an initialized RB can be connected automatically to the respective consequent sets. Influence factors reflect ceteris paribus reasoning and are represented by integers going from –3 to 3 (-3 means strong negative influence, zero means no influence and 3 means strong positive influence).

In the automatic RB-connection procedure, each input and output fuzzy term is identified by its relative position (RP) towards the respective mid-term. For example, the linguistic term-set {very low, low, medium, high, very high} is represented as {-2,-1,0,1,2}. Given the user defined influence factors for all inputs, the relative position $RP(B_i)$ of the consequent set in any rule i is found as follows:

$$RP(B_i) = \text{trunc}\left(\frac{1}{3}\sum_{j=1}^{J} inff_j \cdot RP(A_{ij})\right) \wedge RP(B_i) \geq RP(B)_{min} \wedge RP(B_i) \leq RP(B)_{max}. \tag{12}$$

3.3.2. Rule Base Simplifying

Although the previous step already provides a valid FRB, the user can try to improve its performance. The RB-creating process might lead to a larger number of rules than is necessary. Redundant rules occur because of overlearning, when some observations in the TDS have a higher covering degree than the desired one [GORDÓN/HERRERA (1997), p. 391]. The purpose of simplification is to remove these redundant rules by applying genetic operations on a population of encoded RBs. An RB is encoded as a binary string C of length M, which can be regarded as a sequence of switches that either turn a rule on (1) or off (0). A string only containing 1's represents the created RB of the previous step. All other individuals in the population are initialized randomly. The population is of constant size K with $k \in \{1,...,K\}$.

The user may choose between three selection procedures: (i) Stochastic Universal Sampling (SUS) (rang-based), (ii) SUS (fitness-proportional) and (iii) Tournament Selection to determine the individuals for the mating pool. The activation of the elitist selection option guarantees the survival of the fittest individual in the next population. The offspring population is created by the classical binary multipoint crossover and uniform mutation operators. The fitness of one individual C_k is determined by its MSE over the TDS. The lower this error, the better the individual. Since there is a need to fulfill the *completeness* requirement, the fitness value has to be modified in case of a *completeness* violation. We ensure this by requiring that each observation in the TDS has a CV (equation (3)) greater or equal to τ. The *completeness* property for C_k over the complete TDS is defined as *training-set completeness degree*

$$(13) \quad TSCD(C_k, TDS) = \bigcap_{d_t \in TDS} CV_{C_k}(d_t) .$$

The following fitness function penalizes the fitness value if the *training-set completeness degree* is violated:

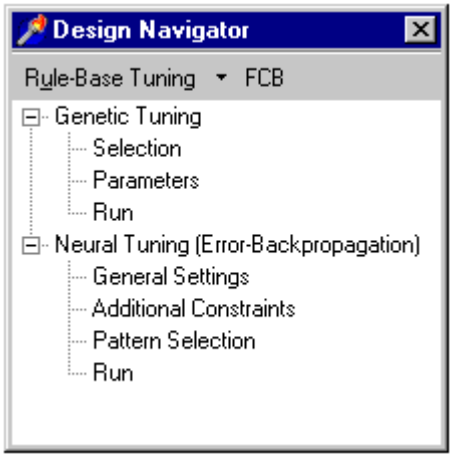
$$(14) \quad F(C_k) = \begin{cases} MSE(C_k) & \text{if } TSCD(C_k, TDS) \geq \tau \\ \frac{1}{2} \sum_{d_t \in TDS} (out_t)^2 & \text{otherwise} \end{cases}$$

It is important to note that GENEFER does not delete any redundant rules. The simplifying procedure attempts to improve the performance of a previously created FRB by switching single rules on and off. As a result, we obtain a binary code of switches that is linked to the generated FRB. This linkage might possibly yield a reduced FRB. Nevertheless, the user is

always able to work with the generated one by (re)-activating all rules (turning on all switches).

3.4. Fuzzy Rule Base Tuning

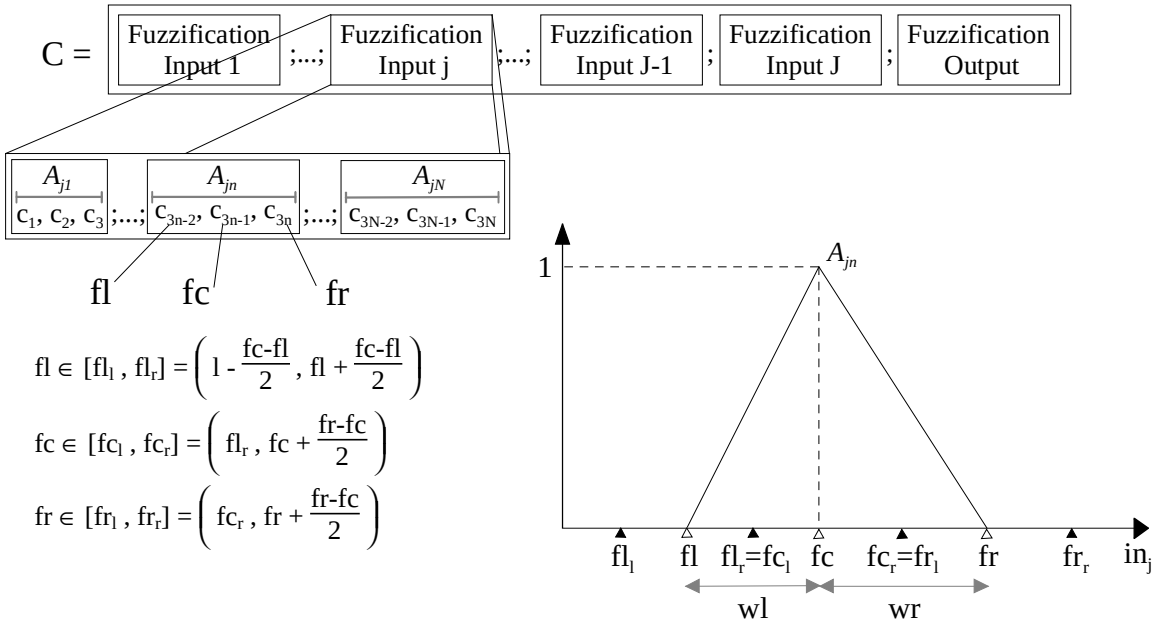
RB-simplifying and FRB-tuning both aim at improving the system's forecast performance. Whereas RB-simplifying uses a given FB in order to refine the RB, the FRB-tuning focuses on adjusting the fuzzy sets' parameters. The current fuzzification of input(s) and output might not be as suitable as possible with respect to the goodness of fit. The search space is limited to the fuzzy sets' centres and widths without changing their a-priori defined type and the degrees of granularity. GENEFER offers two popular tuning algorithms for fuzzy systems: genetic and neural tuning (error-backpropagation). With the latter, the FRB is transformed into a Neural-FRB with a given topology. We do not apply explicit structural learning (changing the network's topology) but focus on parametric learning.



4.3.3. Genetic tuning

The genetic approach requires a suitable representation of the input and output fuzzification. We encode the complete FB as a sequence of real 3-tuples such as (fl, fc, fr) with fl (fr) as a fuzzy set's left (right) border and fc as its centre. The complete encoding C of all fuzzified variables (= FB) is the object to be genetically modified. The encoded fuzzification of one input variable j contains $3 \cdot N_j$ elements c, so that the encoded FB is a sequence of $3 \cdot N_j \cdot N_{out} \cdot J$ elements. The encoded current fuzzification, as well as randomly initialized individuals, form the starting population for the first generation t. Initialization and genetic operations take place in a specified search space. The structure of C, as well as the search space for fuzzy parameter adaptation, is shown in figure 3.

Figure 3: Genetic Tuning of FB



The genetic operators need to be adapted to real coded individuals. We apply non-uniform mutation and Max-Min arithmetical crossover to create individuals for the next generation t+1. If an individual C is randomly selected for mutation, one of its elements c (one of the two borders or a center) is identified to be mutated. A random binary number α determines whether to increase or decrease the value of c. Assume that the selected element c_n to be mutated to c_n' is a center fc then

$$(15) \quad c_n' = \begin{cases} c_n + \Delta(t, fc_r - c_n) & \text{if } \alpha = 1 \\ c_n - \Delta(t, c_n - fc_l) & \text{if } \alpha = 0 \end{cases} .$$

The result of the function $\Delta(t, y)$ is a value in the range [0, y] and the probability of the result's being close to 0 increases in t according to

$$(16) \quad \Delta(t, y) = y\,(1 - r^{(1 - \frac{t}{T})^b}).$$

If a pair of individuals (C_{P1}, C_{P2}) in the current population is selected for crossover, four offspring are created according to

$$(17) \quad \begin{aligned} &C_{1t+1} = a\,C_{P1} + (1-a)\,C_{P2} \\ &C_{2t+1} = a\,C_{P2} + (1-a)\,C_{P1}, C_{3t+1} \quad \text{with } c_3^{t+1} = \min(c_{P1}, c_{P2}) \quad \forall\, c_{P1} \in C_{P1} \text{ and } c_{P2} \in C_{P2} \\ &C_{4t+1} \quad \text{with } c_4^{t+1} = \max(c_{P1}, c_{P2}) \quad \forall\, c_{P1} \in C_{P1} \text{ and } c_{P2} \in C_{P2} \ . \end{aligned}$$

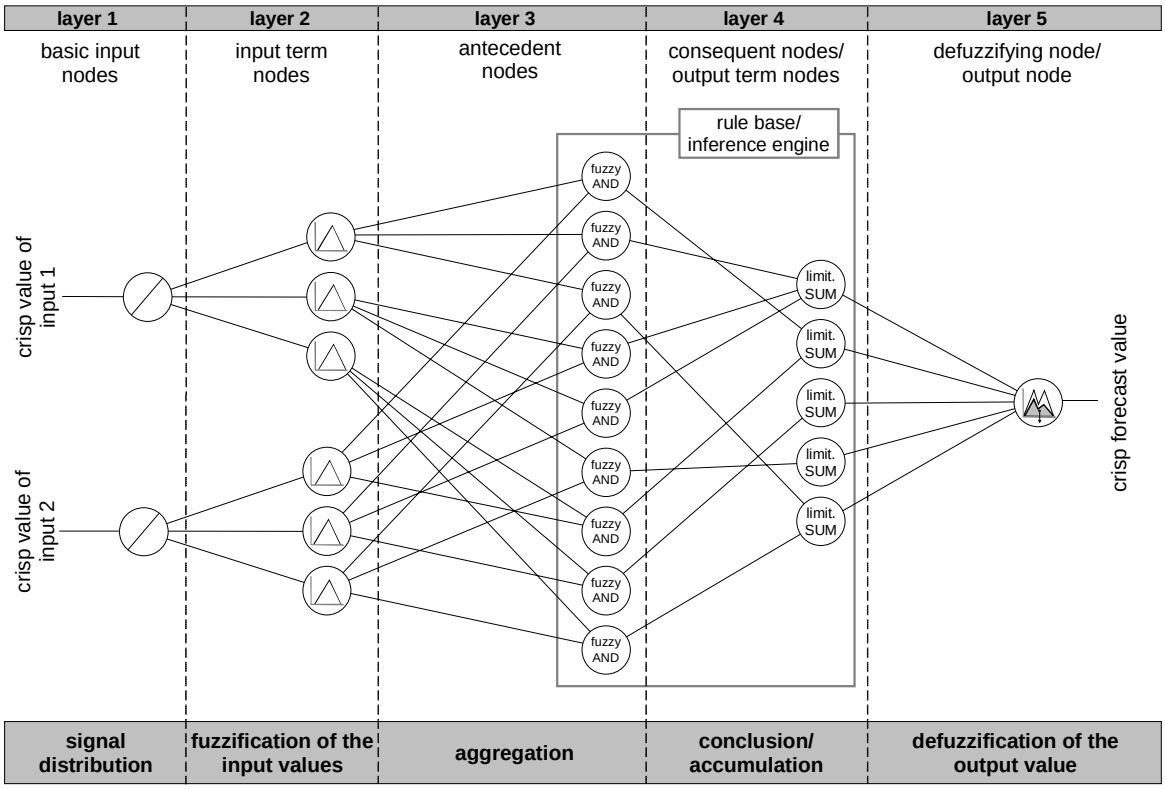
The parameter a is a constant in the range [0,1] and has to be set by the user. To keep the size of the population unchanged, only the two best of the four offspring are copied in the next population. We use the same fitness function as in the previous simplification step for

evaluating the population. Since each individual C represents a complete FB, it is linked to the created (or simplified) RB to determine the FRB's output for MSE calculation.

4.3.4. Neural Tuning (Error-Backpropagation)

The neural FRB-tuning option makes use of the fact that both fuzzy and neural systems are based on a distributed-knowledge representation. This allows to transform a fuzzy system into an equivalent neural network and hence to apply neural learning procedures to fuzzy systems. For this purpose we interpret the FRB as a hybrid neural-fuzzy system (connectionist fuzzy control system) according to the basic technology presented by Lin/Lee (1991) and Lin (1994).[10] This approach uses a layered feed-forward neural network with a total of five layers (see Figure 4). Each layer enacts a specific function in the fuzzy inference process.

Figure 4: Neural fuzzy system with two input variables



The nodes in layer one (basic input nodes) are sensors to the outside world. Their task is to receive the crisp values of input variables and transmit them to the appropriate nodes in layer two (input-term nodes). The input-term nodes carry out the fuzzification function for each input. Every basic input node is connected with all input-term nodes of the respective input variable, which represent the different linguistic terms. The parameters used for characterizing

the membership functions (centers and widths) can be interpreted as link weights between layer one and two. After calculating the degrees of membership for all linguistic input terms the layer two nodes propagate this result to the next layer, whose nodes represent the antecedents of the RB. Each of them computes the degree of activation of the respective rule by means of the fuzzy AND operator. All cross term combinations between all inputs are represented in the aggregation layer, so the number of links of each conditional node to the anterior nodes equals the number of inputs. Since the aggregation procedure works with unweighted input data (i.e. degrees of membership of the concerned terms) the link weights between layer two and three are constant and equal to one. Each node at layer four (consequent nodes in the conclusion/accumulation layer) corresponds to one linguistic term of the output variable (output term nodes). Each of these nodes receives the degrees of application of those conditional nodes that point at the respective consecutive term represented by the node considered at layer four. Optionally the degrees of activation can be weighted by means of link weights between layers three and four. A link weight of zero means that the respective rule has been deactivated. The third and fourth layers constitute the connectionist inference engine that embodies the complete RB of the equivalent fuzzy system. The single node in the fifth layer (output node) defuzzifies the fuzzy inference result and delivers the crisp forecast value. The link weights between layers four and five represent the centers and widths of the fuzzy sets that represent the linguistic terms of the output variable.

The efficiency of a neural network with given topology and node functionality depends only upon the values of the link weights that determine how the node output of layer s is propagated to the subsequent nodes in layer s+1. The knowledge of a neural network is therefore embodied in the values of the link weights. The fact that the complete functionality of the fuzzy inference process is represented equivalently by the neural network allows the application of neural learning methods for FRB-tuning. GENEFER's neural learning method is a modified error-backpropagation procedure (MEBP). The starting point of the learning process is the mean squared error over the TDS:

$$(18) \qquad MSE = \frac{1}{2}\frac{1}{T}\sum_{t=1}^{T}(outFRB(in_t) - out_t)^2.$$

The learning procedure aims at minimizing the error function (18) by finding the weight vector that minimizes MSE. The underlying idea of error-backpropagation is that all nodes of

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[10] A good overview of hybridizing neural and fuzzy technologies is given by Nauck/Klawonn/Kruse (1994), pp. 231 ff. A very concise description of the basic concepts of neural networks can be found in Buckley/Feuring

the network (not only the output node) are responsible for the network error on account of their influence on the signal propagated through the network. During the training phase the signal's direction is reversed, so that each learning round starts at the output node with the network error being fed into the network. This error signal is then backpropagated layer by layer until it reaches the basic input nodes. The global network error is thereby distributed over all relevant nodes. Since each node's signal can only be changed by adjusting its link weights, these are the object of the learning process. The adjustment of each weight w is proportional to its marginal influence on the network error. This method implies a linear approximation of the error function in the environment of the current weight values:

$$(19) \qquad \Delta w = -\mu \frac{\partial MSE}{\partial w}.$$

If we plot the network error as a function of all link weights we get a mountain-like error surface in which (19) describes the steepest descent with the exogenous learning rate μ determining the step size. We omit further details and formulas of the parameter-adjustment procedures here [KOOTHS (1998), section 2.5.3.4], but it should be noticed that they depend on the fuzzy types, the fuzzy AND operator, and the rule weighting option.

The neural learning operations described so far reflect the standard error-backpropagation algorithm (EBP), whose results do not necessarily account for the specific needs of a fuzzy rule base. Sometimes, it is suitable to exclude the adaptation of certain parameters or to restrict their adjustment ranges to keep the whole fuzzy system in a sound state. Otherwise it could happen that the EBP-algorithm ruffles the FB, leaving us with a degenerate system that lacks economic interpretability (e. g., negative or extremely small fuzzy set widths or fuzzy set centers that are difficult to distinguish). In these situations, special MEBP-filters intervene to get a differentiated fuzzification of the relevant crisp data intervals (e. g., by ensuring a minimum overlap of adjacent fuzzy sets). The underlying idea of the adjustment filters is to form the various criteria for a sound FRB and to inhibit the learning process whenever one or more of these criteria is running the risk to be violated.

The MEBP algorithm is especially designed for applying a given FRB to observations that might differ considerably from the training data set. In this case GENEFER finds itself on „virgin soil", which might be due to a radical change within the economic system. Virgin soil appears whenever the fuzzy sets do not cover the relevant crisp interval in an adequately

(1999), ch. 3. For details in neural technologies see Hecht-Nielsen (1991).

differentiated way and/or whenever the relevant crisp values lie in the border regions of the fuzzified interval.

3.5. Adaptability of the system in a changing environment

This section is about GENEFER's application in an economic simulation. As we have mentioned in section 2, there is a need to guarantee the system's adaptability to a changing environment due to the learning ability of agents. The major steps in the design process reveal some useful procedures that can be used for our purposes here as well. Nevertheless GENEFER's spectrum must be extended. The software must allow for *exploitation* of agents' existing knowledge as well as for *exploration* of new knowledge if the observed data conflict with the agents' expectations.

The two tuning procedures in the section above could serve as means for *exploitation* of knowledge. For example, an agent who is used to an inflation rate between 1% and 2% will (sooner or later) change his opinion about the meaning of 'high', if the inflation rate exceeds his experienced top level. The tuning routines adapt the fuzzification of input(s) and output according to a changing economic environment. They check for the existence of a better FB, one that leads to an improved forecast performance. The user specifies a time interval in which no tuning occurs. If he wants to prohibit tuning, he will set this interval equal to the simulation interval; if he wants agents to learn continuously, the interval is set to one.

In contrast to this, *exploration* procedures search for new rules without adjusting the FB. As mentioned above, the generation process creates an FRB, which covers all observations in the TDS but cannot guarantee to provide a fuzzy rule for all possible values of input-output combinations. In the case of a missing rule or poor covering of existing rules[11], GENEFER offers two ways to explore the rule search space. The first one is continuously to replace an agent's FRB after a user-specified number of simulation periods. GENEFER would therefore generate a new FRB on (i) an enlarged TDS including all new observations or (ii) a TDS of constant size including a user specified number of observations (moving window). The technical realization might exceed computational capacity when the number of agents increases or when they are forced to replace their FRB quite frequently. Additionally, one cannot exclude the case of false inference if an uncovered observation occurs during the interval of unchanged FRB. For that reason, we introduce a second procedure that allows one to generate a new rule in case of a new observation for which no forecast value can be

[11] A poor covering may occur with gaussian fuzzy sets, which always yield positive membership values but not necessarily significantly different from zero.

inferred. If, for example, there has never been an observation of low inflation rate differential and high unemployment rate differential, there will probably be no rule covering such an observation. What should an agent expect about the exchange rate (assuming that to be the variable)? Will he throw dice to find out? Not likely. We assume that the agent tries to derive a new rule (and therefore a new forecast result) by using his current knowledge. Let us consider an agent with the FRB in Figure 1 who experiences a low inflation rate differential and a high unemployment differential. Let us also assume that none of his rules covers the observed data, so that no result can be inferred. The agent is assumed to compare the existing rules' antecedents with the observed input data to identify the most similar ones. If we index the fuzzy sets according to their position, we can encode the FRB as

IF	Inflation rate differential	AND	Unemployment rate differential	THEN	€/USD
	1		3		2
	1		5		3
	2		1		1
	3		5		3
	1		2		1
	3		3		2
	3		1		2
	2		2		1
	2		3		2
	3		3		2

We compute the sum of the squared distances between each index of the new antecedent (1,4) and the ones for all antecedents in the FRB above for each rule. The lowest value yields the rule(s) that we regard as most similar. These are the first two in our example ($0^2 + 1^2 = 1$). The agent has to decide whether to expect the first rule's consequent or the second one's. We calculate the mean value and round the result, which yields 'high'. Thus the following rule is added to the agent's current FRB.

	1		4		3

If the observed output at the end of this simulation period matches the newly appended rule's consequent, it will be kept unchanged. If this is not the case, GENEFER replaces the consequent fuzzy set by the one yielding the highest membership value for the observed outcome. Therefore, an agent tends to expect what he has learned from observations so far.

3.6. Key Facts of GENEFER for Economic Model Builders

GENEFER is not limited to a specific type of economic model. It can be implemented in

- *macro-level simulations* using a single FRB for modeling the expectations of the representative agent or the entire market (e. g., Dornbusch's exchange rate model or Laidler's monetarist business-cycle model),
- *micro-level simulations* for multiple-agent modeling with multiple FRBs allowing also for analysis of the interaction of heterogeneous expectations.

The key AI-related features for expectations design are

- three ways to set up a Knowledge Base that consists of fuzzy rules and thus accounts for the agents' bounded rationality. This allows for interpretability of the FRB inference result,
- to modify this FRB to improve its performance with the training data set (offline) or to model the learning processes during a simulation (online). The modification includes the introduction of new fuzzy rules (exploration of knowledge) as well as the tuning of existing rules (exploitation of knowledge),
- obtaining flexibility in adapting the system's behavior to specific purposes. The user can set the agents' inputs by default or let GENEFER learn which inputs are the most significant. He can define each rule in an FRB or let GENEFER learn an RB or even a combination of both. As far as simulation is concerned, the frequency to tune, simplify, or even replace the FRB can be set individually for each agent.

GENEFER comes with COM-server functionality giving access to those FRB-design related methods needed for the implementation of all features mentioned above. The user is therefore free to choose any preferred programming language for his computational economic simulations.

4. Summary and directions for further research

In this paper we propose a new AI-based technique for modeling expectations. This technique combines fuzzy systems as a representation of knowledge bases with Genetic Algorithms (GA) and Artificial Neural Networks as learning operators. We describe their synthesis and present the software GENEFER (GEnetic NEural Fuzzy ExploreR). Economic model builders can implement GENEFER in their simulations via a COM-interface and make use of its fuzzy

inference and learning routines. Nevertheless it may also be used for pure forecasting purposes on empirical data.

We are currently testing GENEFER's performance in forecasting financial time series. The development of self-documenting-business-cycle-indicators is on our research agenda for the near future. GENEFER's modular architecture gives room for further combinations of AI techniques. We plan to apply GA in order to evolve a population of Fuzzy Rule Bases and therefore reduce the amount of exogenous parameters, e.g., the learning rate.

For the download of the software GENEFER as well as additional information including illustrative examples we refer to www.genefer.de.

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